

Curie temperature of the two band double exchange model for manganites

Vasil Michev and Naoum Karchev[*]

Department of Physics, University of Sofia, 1164 Sofia, Bulgaria

We consider two-band double exchange model and calculate the critical temperature in ferromagnetic regime (Curie temperature). The localized spins are represented in terms of the Schwinger-bosons, and two spin-singlet Fermion operators are introduced. In terms of the new Fermi fields the on-site Hund's interactions are in a diagonal form and one accounts for them exactly. Integrating out the spin-singlet fermions we derive an effective Heisenberg type model for a vector which describes the local orientations of the total magnetization. The transversal fluctuations of the vector are the true magnons in the theory, which is a base for Curie temperature calculation. The critical temperature is calculated employing the Schwinger-bosons mean-field theory. While approximate, this technic of calculation captures the essentials of the magnon fluctuations in the theory, and for 2D systems one obtains zero Curie temperature, in accordance with Mermin-Wagner theorem.

PACS numbers: 75.47.Lx, 71.27.+a, 75.30.Ds, 75.50.Cc

I. INTRODUCTION

The manganites are ones of the prominent representatives of strongly correlated systems, where the effect of correlations among electrons plays a crucial role. The growing interest on manganites is mainly due to observation of resistivity changes by many orders of magnitude upon the application of small magnetic fields, an effect that carries the name of "Colossal Magnetoresistance".

The attempts to properly describe the physics of these materials by means of simple perturbative methods typically fail [1]. One have to develop technic of calculation which capture the essential features of the compounds. The double exchange model is a widely used model for manganites [1, 2, 3]. In isolation, the ions of Mn have an active $3d$ -shell with five degenerate levels. The degeneracy is presented due to rotational invariance within angular momentum $l = 2$ subspace. The crystal environment results in a particular splitting of the five d -orbitals (*crystal field splitting*) into two groups: the e_g and t_{2g} states. The electrons from the e_g sector, which form a doublet, are removed upon hole doping. The t_{2g} electrons, which form a triplet, are not affected by doping, and their population remains constant. The Hund rule enforces alignment of the three t_{2g} spins into a $S = 3/2$ state. Then, the t_{2g} sector can be replaced by a *localized spin* at each manganese ion, reducing the complexity of the original five orbital model. The only important interaction between the two sectors is the Hund coupling between localized t_{2g} spins and mobile e_g electrons. A strong impact on the physics of manganites has the static Jahn-Teller distortion which leads to a splitting of the degenerate e_g levels.

The double exchange model has a rich phase diagram, exhibiting a variety of phases, with unusual ordering in the ground states. The procedures followed to obtain the phase diagram in one band model are different: numerical studies [4], dynamical mean field theory [5], and analytical calculations [6, 7], but four phases have been

systematically observed: (i) antiferromagnetism (AF) at a density of mobile electrons $n = 1$, (ii) ferromagnetism (FM) at intermediate electronic densities, (iii) phase separation (PS) between FM and AF phases, and (iv) spin incommensurate (IC) phase at large enough Hund coupling. The competition between spin spiral incommensurate order or phase separation and canted ferromagnetism is also a topic of intensive study [6, 7, 8]. The phase diagram becomes more rich if the orbital degeneracy is accounted for [9, 10].

Double exchange model is also successful in calculating the critical temperature in the ferromagnetic regime. Predictions about Curie temperature in one band double exchange model are made using Monte Carlo technique [4], High-temperature series expansion [11], Dynamical Mean-Field Approximation [12] and standard Mean-Field approach [13]. The most striking feature of the critical temperature as function of fermion density is the symmetry with respect to $n = 0.5$. The Curie temperature is maximized at that point, and maximal value is different within different approaches.

Very recently the critical temperature was calculated using two-band model with account for the Jahn-Teller effect [14, 15]. Again the characteristic feature of the temperature curves as a function of charge carrier density is the symmetry with the respect to $n = 1$. The new assertion is that the Curie temperature increases with increasing the interband hopping [14].

It is impossible to require the theoretically calculated Curie temperature to be in accordance with experimental results. The models are idealized, and they do not consider many important effects: phonon modes, several types of disorder, Coulomb interaction, etc. Because of that it is important to formulate theoretical criteria for adequacy of the method of calculation. In our opinion the calculations should be in accordance with Mermin-Wagner theorem [16]. It claims that in two dimension there is not spontaneous magnetization at non-zero temperature. Hence, the critical temperature should be equal

to zero. It is well known that Monte Carlo method of calculation does not satisfy this criteria [1]. It is difficult within Dynamical Mean-Field theory to make a difference between two dimensional and three dimensional systems. DEM is a good approximation when the dimensionality goes to infinity. This made the analytical methods important even for the assessment of the numerical results.

The paper is organized as follows: In Sec. II, starting from two band double exchange model, we derive an effective Heisenberg like model in terms of vector describing the local orientations of the total magnetization. The transversal fluctuations of the vector are the true magnons in the theory. This is a base for Curie temperature calculation. Sec. III is devoted to phase diagrams of the model in space of Hund's constant and charge carrier density. We calculate spin-stiffness constant as a function of density which is important step towards understanding the Curie temperature behavior as a function of charge carrier density. The results for the Curie temperature are reported in Sec. IV. A summary in Sec. V concludes the paper. Spin-stiffness constant calculations are presented in the Appendix.

II. EFFECTIVE MODEL

We consider a two-band double-exchange model, with Hamiltonian:

$$H = - \sum_{ll' < ij > \sigma} (t_{ll'} c_{il'\sigma}^\dagger c_{jl\sigma} + h.c.) - 2 \sum_{il} J_l \vec{S}_i \cdot \vec{s}_{il} - \mu \sum_{il} c_{il\sigma}^\dagger c_{il\sigma} \quad (\text{II.1})$$

where l, l' are band indexes, i, j are site labels, σ are the spin indices, $c_{il\sigma}^\dagger$ and $c_{il\sigma}$ are creation and destruction operators for mobile electrons, μ is the chemical potential. s_{il} is the spin of the conduction electrons, and \mathbf{S}_i is the spin of the localized electrons. The sums are over all sites of a three-dimensional cubic lattice, and $\langle i, j \rangle$ denotes the sum over the nearest neighbors. We denote $t_{11} \equiv t_1$, $t_{22} \equiv t_2$, and $t_{12} \equiv t_{21} \equiv t'$.

In terms of Schwinger-bosons ($\varphi_{i\sigma}, \varphi_{i\sigma}^\dagger$) the core spin operators have the following representation:

$$\vec{S}_i = \frac{1}{2} \varphi_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} \varphi_{i\sigma'} \quad \varphi_{i\sigma}^\dagger \varphi_{i\sigma} = 2s \quad (\text{II.2})$$

with the Pauli matrices (τ^x, τ^y, τ^z).

The partition function can be written as a path integral over the complex functions of the Matsubara time $\varphi_{i\sigma}(\tau)$ ($\varphi_{i\sigma}^\dagger(\tau)$) and Grassmann functions $c_{il\sigma}(\tau)$ ($c_{il\sigma}^\dagger(\tau)$).

$$\mathcal{Z}(\beta) = \int d\mu (\varphi^+, \varphi, c^+, c) e^{-S} \quad (\text{II.3})$$

with an action given by the expression

$$S = \int_0^\beta d\tau \left[\sum_i (\varphi_{i\sigma}^+(\tau) \dot{\varphi}_{i\sigma}(\tau) + c_{il\sigma}^+(\tau) \dot{c}_{il\sigma}(\tau)) + h(\varphi^+, \varphi, c^+, c) \right] \quad (\text{II.4})$$

where β is the inverse temperature and the Hamiltonian is obtained from equations (II.1) and (II.2) replacing the operators with the functions.

We introduce spin-singlet Fermi fields

$$\Psi_{il}^A(\tau) = \frac{1}{\sqrt{2s}} \varphi_{i\sigma}^+(\tau) c_{il\sigma}(\tau) \quad (\text{II.5})$$

$$\Psi_{il}^B(\tau) = \frac{1}{\sqrt{2s}} [\varphi_{i1}(\tau) c_{il2}(\tau) - \varphi_{i2}(\tau) c_{il1}(\tau)] \quad (\text{II.6})$$

which are $U(1)$ gauge variant with charge -1 and 1 respectively

$$\Psi_{jl}'^A(\tau) = e^{-i\alpha_j(\tau)} \Psi_{jl}^A(\tau) \quad \Psi_{jl}'^B(\tau) = e^{i\alpha_j(\tau)} \Psi_{jl}^B(\tau) \quad (\text{II.7})$$

The equations (II.5) and (II.6) can be regarded as a $SU(2)$ transformation:

$$\Psi_{il\sigma} = g_{i\sigma\sigma'}^+ c_{il\sigma'} \Rightarrow g_i^+ = \frac{1}{\sqrt{2s}} \begin{pmatrix} \varphi_{i1}^+ & \varphi_{i2}^+ \\ -\varphi_{i2} & \varphi_{i1} \end{pmatrix} \quad (\text{II.8})$$

with $\Psi_{il}^A = \Psi_{il1}$ and $\Psi_{il}^B = \Psi_{il2}$.

In terms of the new Fermi fields, electron creation and destruction operators have the form:

$$\begin{aligned} c_{il1} &= \frac{1}{\sqrt{2s}} (\varphi_{i1} \Psi_{il}^A - \varphi_{i2}^+ \Psi_{il}^B) \\ c_{il2} &= \frac{1}{\sqrt{2s}} (\varphi_{i2} \Psi_{il}^A + \varphi_{i1}^+ \Psi_{il}^B) \\ c_{il1}^+ &= \frac{1}{\sqrt{2s}} (\varphi_{i1}^+ \Psi_{il}^{A\dagger} - \varphi_{i2} \Psi_{il}^{B\dagger}) \\ c_{il2}^+ &= \frac{1}{\sqrt{2s}} (\varphi_{i2}^+ \Psi_{il}^{A\dagger} + \varphi_{i1} \Psi_{il}^{B\dagger}) \end{aligned} \quad (\text{II.9})$$

and the spin of the conduction electrons \mathbf{s}_{il} is

$$\mathbf{s}_{il}^\mu = \frac{1}{2} c_{il\sigma}^\dagger \tau_{\sigma\sigma'}^\mu c_{il\sigma'} = \frac{1}{2} \text{O}_i^{\mu\nu} \Psi_{il\sigma}^\dagger \tau_{\sigma\sigma'}^\nu \Psi_{il\sigma'}, \quad (\text{II.10})$$

where

$$\text{O}_i^{\mu\nu} = \frac{1}{2} \text{Tr } g_i^+ \tau^\mu g_i \tau^\nu. \quad (\text{II.11})$$

It is convenient to introduce three basic vectors which depend on the Schwinger-bosons

$$\mathbf{T}_{i\mu}^1 = \text{O}_i^{\mu 1} \quad \mathbf{T}_{i\mu}^2 = \text{O}_i^{\mu 2} \quad \mathbf{T}_{i\mu}^3 = \text{O}_i^{\mu 3}, \quad (\text{II.12})$$

where $\mathbf{T}_i^3 = \frac{1}{s} \mathbf{S}_i$. Then, the spin of the electrons can be represented as a linear combination of three vectors \mathbf{S}_j , $\mathbf{P}_j = \mathbf{T}_j^1 + i\mathbf{T}_j^2$ and $\mathbf{P}_j^\dagger = \mathbf{T}_j^1 - i\mathbf{T}_j^2$

$$\begin{aligned} \mathbf{s}_{il} &= \frac{1}{2s} \mathbf{S}_i (\Psi_{il}^A + \Psi_{il}^A - \Psi_{il}^{B\dagger} + \Psi_{il}^B) \\ &+ \frac{1}{2} \mathbf{P}_i \Psi_{il}^{B\dagger} \Psi_{il}^A + \frac{1}{2} \mathbf{P}_i^\dagger \Psi_{il}^A \Psi_{il}^B. \end{aligned} \quad (\text{II.13})$$

The basic vectors satisfy the relations $\mathbf{S}_i^2 = s^2$, $\mathbf{P}_i^2 = \mathbf{P}_i^{+2} = \mathbf{S}_i \cdot \mathbf{P}_i = \mathbf{S}_i \cdot \mathbf{P}_i^+ = 0$, and $\mathbf{P}_i^+ \cdot \mathbf{P}_i = 2$. Using the expression (II.13) for the spin of itinerant electrons, the total spin of the system

$$\mathbf{S}_i^{\text{tot}} = \mathbf{S}_i + \mathbf{s}_{i1} + \mathbf{s}_{i2} \quad (\text{II.14})$$

can be written in the form

$$\begin{aligned} \mathbf{S}_i^{\text{tot}} = & \frac{1}{s} \left[s + \frac{1}{2} \sum_l (\Psi_{il}^{A+} \Psi_{il}^A - \Psi_{il}^{B+} \Psi_{il}^B) \right] \mathbf{S}_i + \\ & \frac{1}{2} \mathbf{P}_i \sum_l \Psi_{il}^{B+} \Psi_{il}^A + \frac{1}{2} \mathbf{P}_i^+ \sum_l \Psi_{il}^{A+} \Psi_{il}^B \quad (\text{II.15}) \end{aligned}$$

The gauge invariance imposes the conditions $\langle \Psi_{il}^{A+} \Psi_{il}^B \rangle = \langle \Psi_{il}^{B+} \Psi_{il}^A \rangle = 0$. As a result, the dimensionless magnetization per lattice site $\langle (S_i^{\text{tot}})^z \rangle$ reads

$$\langle (S_i^{\text{tot}})^z \rangle = \frac{1}{s} \left[s + \frac{1}{2} \sum_l \langle (\Psi_{il}^{A+} \Psi_{il}^A - \Psi_{il}^{B+} \Psi_{il}^B) \rangle \right] \langle \mathbf{S}_i^z \rangle \quad (\text{II.16})$$

Let us average the total spin of the system (Eq. II.15) in the subspace of the itinerant electrons $\langle \mathbf{S}_i^{\text{tot}} \rangle_f = \mathbf{M}_i$. The vector \mathbf{M}_i identifies the local orientation of the total magnetization. Accounting for the gauge invariance, one obtains the following expression for \mathbf{M}_i

$$\langle \mathbf{S}_i^{\text{tot}} \rangle_f = \mathbf{M}_i = \frac{M}{s} \mathbf{S}_i \quad (\text{II.17})$$

where

$$M = s + \frac{1}{2} \sum_l \langle (\Psi_{il}^{A+} \Psi_{il}^A - \Psi_{il}^{B+} \Psi_{il}^B) \rangle \quad (\text{II.18})$$

can be thought of as an "effective spin" of the system ($\mathbf{M}_i^2 = M^2$). Now, if we use Holstein-Primakoff representation for the vectors \mathbf{M}_j

$$\begin{aligned} M_j^+ &= M_{j1} + iM_{j2} = \sqrt{2M - a_j^+ a_j} a_j \\ M_j^- &= M_{j1} - iM_{j2} = a_j^+ \sqrt{2M - a_j^+ a_j} \\ M_j^3 &= M - a_j^+ a_j \end{aligned} \quad (\text{II.19})$$

the bose fields a_j and a_j^+ are the **true magnons** in the system. In terms of the true magnons the Schwinger-bosons (II.2) have the following representation

$$\varphi_{i1} = \sqrt{2s - \frac{s}{M} a_i^+ a_i}, \quad \varphi_{i2} = \sqrt{\frac{s}{M}} a_i. \quad (\text{II.20})$$

Replacing in Eqs. (II.5) and (II.6) for the spin-singlet Fermions and keeping only the first two terms in $1/M$ expansion $\sqrt{1 - \frac{1}{2M} a_i^+ a_i} \simeq 1 - \frac{1}{4M} a_i^+ a_i + \dots$ we obtain

$$\Psi_{il}^A = c_{il1} + \frac{1}{\sqrt{2M}} a_i^+ c_{il2} - \frac{1}{4M} a_i^+ a_i c_{il1} + \dots \quad (\text{II.21})$$

$$\Psi_{il}^B = c_{il2} - \frac{1}{\sqrt{2M}} a_i c_{il1} - \frac{1}{4M} a_i^+ a_i c_{il2} + \dots \quad (\text{II.22})$$

The equations (II.21) and (II.22) show that the singlet fermions are electrons dressed by a virtual cloud of repeatedly emitted and reabsorbed magnons.

An important advantage of working with A and B fermions is the fact that in terms of these spin-singlet fields the spin-fermion interaction is in a diagonal form, the spin variables (magnons) are removed, and one accounts for it exactly:

$$\sum_{il} \vec{S}_i \cdot \vec{s}_{il} = \frac{s}{2} \sum_{il} (\Psi_{il}^{A+} \Psi_{il}^A - \Psi_{il}^{B+} \Psi_{il}^B) \quad (\text{II.23})$$

Replacing all this into the action (II.4), we can rewrite it as a function of the Schwinger-bosons and spin-singlet fermions. The resulting action is quadratic with respect to the spin-singlet fermions and one can integrate them out. The effective Hamiltonian of the theory, in Gaussian approximation, is given by:

$$h_{\text{eff}} = \rho \sum_{\langle ij \rangle} (a_i^+ a_i + a_j^+ a_j - a_i^+ a_j - a_j^+ a_i) \quad (\text{II.24})$$

where ρ is the spin stiffness (A.42). Detailed calculation are given in the appendix. Based on the rotational symmetry, one can supplement the Hamiltonian (Eq. II.24) up to an effective Heisenberg like Hamiltonian, written in terms of the vectors \mathbf{M}_i

$$h_{\text{eff}} = -J \sum_{\langle ij \rangle} \mathbf{M}_i \cdot \mathbf{M}_j \quad (\text{II.25})$$

where $J = \rho/M$. The ferromagnetic phase is stable if the effective exchange coupling constant is positive $J > 0$.

It is important to highlight the difference between the above effective theory (II.25) and Ruderman-Kittel-Kasuya-Yosida (RKKY) theory. The RKKY effective Hamiltonian is an effective Heisenberg like Hamiltonian in terms of core spins \mathbf{S}_i , obtained averaging in the subspace of the itinerant electrons [17]. The subtle point is that if we use a Holstein-Primakoff representation for the localized spins \mathbf{S}_i , the creation and annihilation bose operators do not describe the true magnon of the system [18]. The true magnons are transversal fluctuations corresponding to the total magnetization which includes both the spins of localized and delocalized electrons. Therefore the RKKY validity condition requires small Hund's coupling, and small density of charge carriers, which in turn means that the magnetization of the mobile electrons is inessential. In contrast of RKKY theory the effective model (II.25) is written in term of vectors \mathbf{M}_i which describe the local orientations of the total magnetization, and the bose operators in (II.19) are the true magnons in the theory. This is essential when one calculates the Curie temperature. The effective model (II.25) is obtained integrating out the spin-singlet fermions (II.5) and (II.6). In terms of these fermions the Hund's interaction is in a diagonal form and we account for it exactly. Hence, the effective theory (II.25) is valid for arbitrary values of Hund's constants and for all densities of charge carriers.

III. PHASE DIAGRAMS

Here we illustrate some of the features of our model, namely the phase diagrams and how they change when we vary the model's parameters. Since calculating T_c is closely related to calculating spin stiffness ρ , we have examined the behavior of ρ as a function of electron density in details.

The physics of the model depends on dimensionless parameters J_1/t_1 , J_2/t_1 , t_2/t_1 and t'/t_1 . Throughout this chapter we fix the scale setting $t_1 = t_2 = 1$, and use J_1 , J_2 and t' as a free parameters of the model. Also in this section $t' = 0.5$, and since we are describing manganite materials, we have set $s = 3/2$. The phase diagrams on Fig.1 are constructed by plotting the curve $\rho = 0$ in coordinate system of carrier density n and J_2/t_1 for fixed ratio J_1/J_2 . Regions where $\rho > 0$ correspond to the ferromagnetic phase (FM), while those with $\rho < 0$ are denoted here simply as non-FM ones, since describing all possible phases is not the purpose of this paper.

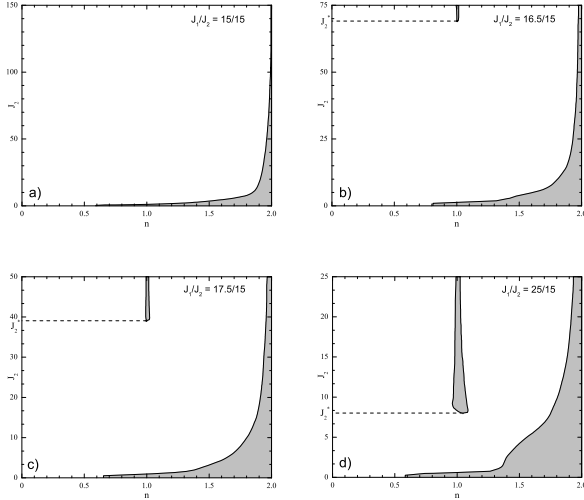


FIG. 1: Phase diagrams for the four cases under consideration: a) $J_1 = J_2 = 15$, b) $J_1 = 16.5$, $J_2 = 15$, c) $J_1 = 17.5$, $J_2 = 15$ and d) $J_1 = 25$, $J_2 = 15$. White regions correspond to FM phase, grayed ones to FM instability.

We consider four different cases for the ratio J_1/J_2 , namely $J_1/J_2 = 15/15$, where the bands are degenerated, and three cases with increasing bands' splitting $J_1/J_2 = 16.5/15$, $J_1/J_2 = 17.5/15$ and $J_1/J_2 = 25/15$.

When the bands are split, we observe an island of ferromagnetic instability around the line $n = 1$. The lowest point of the island is denoted by J_2^* . Increasing the ratio J_1/J_2 increases the island by lowering the value of J_2^* (see Fig.1). For the degenerated bands ($J_1/J_2 = 1$), we have not observed the island of ferromagnetic instability, up to values as large as $J_2 = 300$.

The value of the spin stiffness constant ρ depends on

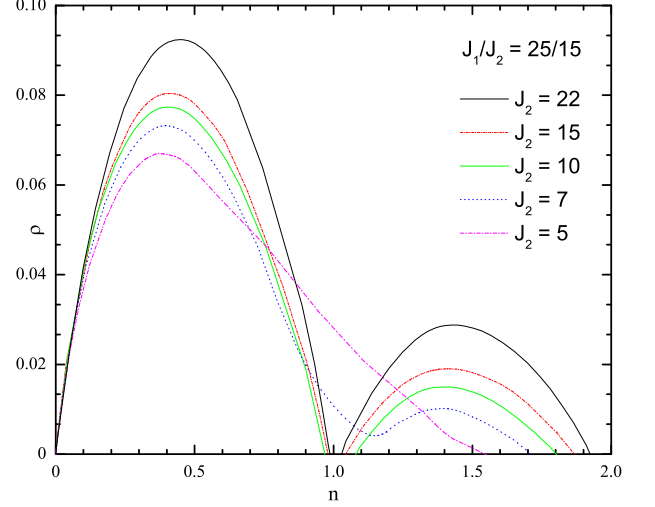


FIG. 2: (color online) ρ as a function of n for the forth case $J_1/J_2 = 25/15$, corresponding to fig. 1d.

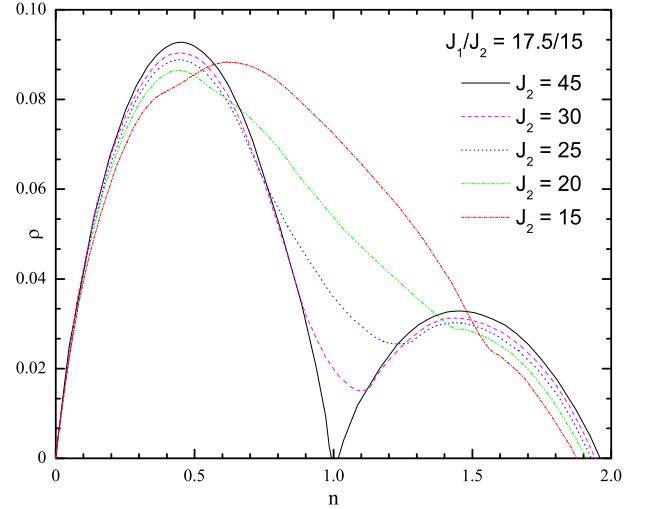


FIG. 3: (color online) ρ as a function of n for the third case $J_1/J_2 = 17.5/15$, corresponding to fig. 1c.

the point (n, J_2) in the phase diagram, for fixed ratio J_1/J_2 . When the point approaches the boundary of the ferromagnetic phase, the value of ρ decreases and reaches zero on the boundary (by the definition of the boundary). We have calculated the spin stiffness constant ρ as a function of n , for fixed J_2 at zero temperature. There are three distinctive cases: $J_2 > J_2^*$, $J_2 \lesssim J_2^*$ and $J_2 \ll J_2^*$.

For the first one, the presence of ferromagnetic instability near $n = 1$ results in a function $\rho(n)$, which consists of two pieces, one for n in the interval $(0, 1)$, and another for n in the interval $(1, 2)$. For the ratio $J_1/J_2 = 25/15$

(Fig.2), such are the curves corresponding to $J_2 = 22$, $J_2 = 15$ and $J_2 = 10$; for $J_1/J_2 = 17.5/15$ (Fig.3) the curve corresponding to $J_2 = 45$; and for $J_1/J_2 = 16.5/15$ (Fig.4) the one corresponding to $J_2 = 75$. The important characteristic of all these curves is the existence of two maxima, one within interval $(0, 1)$, and another one within interval $(1, 2)$. The global maximum is within the interval $(0, 1)$, which is result of the presence of two phase boundaries in the other interval. In the case of degenerated bands, the absence of island of instability leads to the absence of such type of function $\rho(n)$.

For the second one, J_2 is very close to J_2^* , hence near $n = 1$ the spin stiffness constant is very small. As a result $\rho(n)$ is a function with two maxima and one minimum. For the ratio $J_1/J_2 = 25/15$ (Fig.2), such is the curve corresponding to $J_2 = 7$; for $J_1/J_2 = 17.5/15$ (Fig.3) the curves corresponding to $J_2 = 30$ and $J_2 = 25$; and for $J_1/J_2 = 16.5/15$ (Fig.4) the ones corresponding to $J_2 = 60$ and $J_2 = 50$. The minimal value of the function decreases when J_2 approaches J_2^* . Again, in the special case of degenerated bands, there is no such a curve.

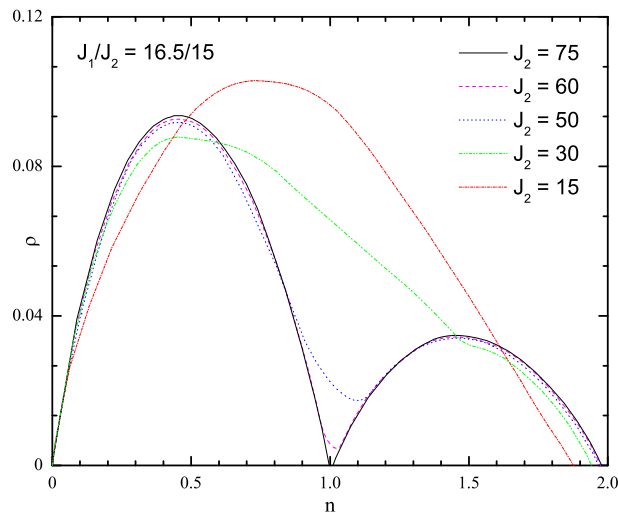


FIG. 4: (color online) ρ as a function of n for the second case $J_1/J_2 = 16.5/15$, corresponding to fig. 1b.

For the third case, $J_2 \ll J_2^*$, we are well below the island of ferromagnetic instability, and the function $\rho(n)$ has only one maximum within the interval $(0, 1)$. For the ratio $J_1/J_2 = 25/15$ (Fig.2), such is the curve corresponding to $J_2 = 5$; for $J_1/J_2 = 17.5/15$ (Fig.3) the curves corresponding to $J_2 = 20$ and $J_2 = 15$; and for $J_1/J_2 = 16.5/15$ (Fig.4) the ones corresponding to $J_2 = 30$ and $J_2 = 15$. In the case of degenerated bands, for all values of J_2 the curves are of this type, because of the absence of instability island (Fig.5).

It is widely known fact, that near $n = 2$ the system is ferromagnetically unstable and the spin stiffness constant approaches zero when the carrier density approaches two.

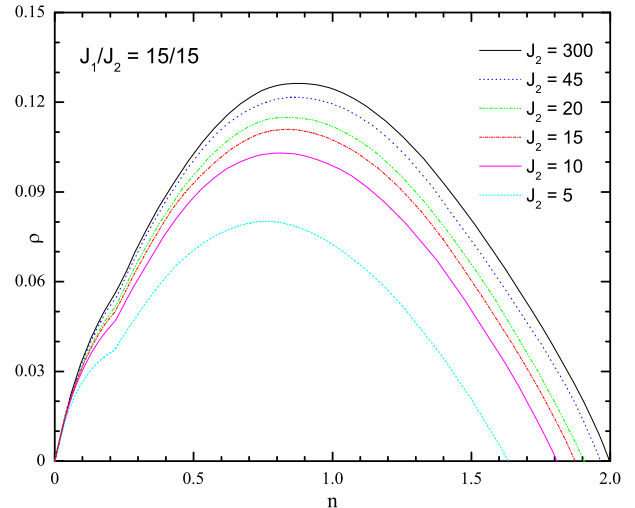


FIG. 5: (color online) ρ as a function of n for the degenerated case $J_1/J_2 = 15/15$, corresponding to fig. 1a.

One can see from the phase diagrams (Fig.1), that with decreasing of J_2 , the ferromagnetic instability sets in for smaller value of n . As a result we obtain that with decreasing J_2 , the value of n for which $\rho(n) = 0$ decreases. This is best observed in the case of degenerated bands, where we have examined broader set of values for J_2 . With increasing of J_2 , the point $\rho = 0$ gets closer to $n = 2$ and the curve becomes more symmetric.

Since the maximum value of the Curie temperature corresponds to the maximum value of the spin stiffness constant, it is important to see how this value depends on the ratio J_1/J_2 for fixed J_2 . We choose the case $J_2 = 15$ (red lines in the figures), to compare our results with the results in [14]. Increasing J_1/J_2 increases the ferromagnetic instability, see phase portraits Fig. 1, lowering the point J_2^* which in turn leads to decrease of the maximum value of ρ and qualitative changes in its behavior as a function of density n .

IV. CURIE TEMPERATURES

To calculate the Curie temperature we utilize the Schwinger-bosons mean-field theory [19, 20]. The advantage of this method of calculation is that for 2D systems one obtains zero Curie temperature, in accordance with Mermin-Wagner theorem [16]. So, while approximate, this technique of calculation captures the essentials of the magnon fluctuations in the theory.

To proceed, we represent the effective spin vector \mathbf{M}_i by means of Schwinger bosons $\phi_{i\sigma}, \phi_{i\sigma}^\dagger$,

$$M_i^\nu = \frac{1}{2} \sum_{\sigma\sigma'} \phi_{i\sigma}^\dagger \tau_{\sigma\sigma'}^\nu \phi_{i\sigma'}, \quad \phi_{i\sigma}^\dagger \phi_{i\sigma} = 2M \quad (\text{IV.1})$$

Next we use the identity

$$\mathbf{M}_i \cdot \mathbf{M}_j = \frac{1}{2} (\phi_{i\sigma}^+ \phi_{j\sigma}) (\phi_{j\sigma'}^+ \phi_{i\sigma'}) - \frac{1}{4} (\phi_{i\sigma}^+ \phi_{i\sigma}) (\phi_{j\sigma'}^+ \phi_{j\sigma'}) \quad (\text{IV.2})$$

and rewrite the effective Hamiltonian in the form

$$h_{\text{eff}} = -\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{i\sigma}^+ \phi_{j\sigma}) (\phi_{j\sigma'}^+ \phi_{i\sigma'}) \quad (\text{IV.3})$$

where the second term in (IV.2) is equal to the constant M^2 , because of the constraint (IV.1), and we drop it. To ensure the constraint (IV.1) we introduce a parameter (λ) and add a new term to the effective Hamiltonian (IV.3)

$$\hat{h}_{\text{eff}} = h_{\text{eff}} + \lambda \sum_i (\phi_{i\sigma}^+ \phi_{i\sigma} - 2M) \quad (\text{IV.4})$$

We treat the four-boson interaction within Hartree-Fock approximation. The Hartree-Fock hamiltonian which corresponds to the effective hamiltonian (IV.4) reads

$$h_{\text{H-F}} = \frac{J}{2} \sum_{\langle ij \rangle} \bar{u}_{ij} u_{ij} - \frac{J}{2} \sum_{\langle ij \rangle} [\bar{u}_{ij} \phi_{i\sigma}^+ \phi_{j\sigma} + u_{ij} \phi_{j\sigma}^+ \phi_{i\sigma}] + \lambda \sum_i (\phi_{i\sigma}^+ \phi_{i\sigma} - 2M) \quad (\text{IV.5})$$

where \bar{u}_{ij} (u_{ij}) are Hartree-Fock parameters to be determined self-consistently. We are interested in real parameters which do not depend on the lattice sites, $u_{ij} = \bar{u}_{ij} = u$. Then in momentum space representation, the Hamiltonian (IV.5) has the form

$$h_{\text{H-F}} = \frac{3J}{2} N u^2 - 2\lambda M N + \sum_k \varepsilon_k \phi_k^+ \phi_k \quad (\text{IV.6})$$

where N is the number of lattice sites and

$$\varepsilon_k = \lambda - J u (\cos k_x + \cos k_y + \cos k_z) \quad (\text{IV.7})$$

is the dispersion of the ϕ_k -boson (spinon).

The free energy of a theory with Hamiltonian (IV.6) is

$$F = \frac{3J}{2} u^2 - 2\lambda M + \frac{2T}{N} \sum_k \ln \left(1 - e^{-\frac{\varepsilon_k}{T}} \right) \quad (\text{IV.8})$$

where T is the temperature. The self-consistent equations for parameters u and λ are

$$\frac{\partial F}{\partial u} = 0 \quad \frac{\partial F}{\partial \lambda} = 0 \quad (\text{IV.9})$$

We obtain a system of two equations

$$u = \frac{2}{3} \frac{1}{N} \sum_k n_k (\cos k_x + \cos k_y + \cos k_z) \quad (\text{IV.10})$$

$$M = \frac{1}{N} \sum_k n_k \quad (\text{IV.11})$$

where n_k is the bose function

$$n_k = \frac{1}{e^{\frac{\varepsilon_k}{T}} - 1} \quad (\text{IV.12})$$

To ensure correct definition of the bose theory (IV.6) we have to make some assumptions for the parameter λ . For that purpose its convenient to represent it in the form

$$\lambda = 3Ju + \mu Ju \quad (\text{IV.13})$$

In terms of the new parameter μ , the bose dispersion is

$$\varepsilon_k = Ju (3 - \cos k_x - \cos k_y - \cos k_z + \mu) \quad (\text{IV.14})$$

and the theory is well defined for $\mu \geq 0$.

We find the parameters μ and u solving the equations (IV.10-IV.11). For high enough temperatures both $\mu(T) > 0$ and $u(T) > 0$ and the excitation is gapped. It is the spinon excitation in the theory in the paramagnetic phase. Decreasing the temperature leads to decrease of $\mu(T)$. At temperature T_C it becomes equal to zero and long-range excitation emerges in the spectrum. Hence the temperature at which μ reaches zero is the Curie temperature. We set $\mu = 0$ in the system of equations (IV.10-IV.11) and obtain a system of two equations for the Curie temperature T_C and the parameter u , which is the renormalization of the exchange constant at Curie temperature.

$$u = \frac{2}{3} \frac{1}{N} \sum_k \frac{\cos k_x + \cos k_y + \cos k_z}{e^{\frac{\rho u}{MT_C} (3 - \cos k_x - \cos k_y - \cos k_z)} - 1} \quad (\text{IV.15})$$

$$M = \frac{1}{N} \sum_k \frac{1}{e^{\frac{\rho u}{MT_C} (3 - \cos k_x - \cos k_y - \cos k_z)} - 1}$$

To calculate T_C we solve the above system of equations, with $M(T)$ and $\rho(T)$ calculated from equations (II.18) and (A.42) respectively.

The results for the Curie temperature T_C as a function of charge density n are plotted on figures 6-9, for two different values of interband hopping parameter t' . In all cases we have set $t_1 = t_2 = 1$, and consider a theory with $s = 3/2$ for the core spins. We want to compare our results with the results in [14], so we have set $J_2 = 15$ and $t' = 0.5$. Since our method of calculating T_C involves $\rho(T)$, the resulting curves are very similar to the ones in section III (Fig.2-5), and bear their characteristics. As above we have three different choices for the parameters: $J_2 > J_2^*$, $J_2 \lesssim J_2^*$ and $J_2 \ll J_2^*$. For the biggest ratio we consider $J_1/J_2 = 25/15$ (see Fig.6), we have a two-piece function, since our chosen value $J_2 > J_2^*$. With decreasing the ratio, the point J_2^* moves to higher values and J_2 becomes smaller than J_2^* . As a result the curves we obtained have only one maximum. It is important to note, that even for the degenerated case $J_1/J_2 = 1$, the point at which T_C reaches zero is smaller than $n = 2$ unlike the results in [14]. This can be also seen from the phase

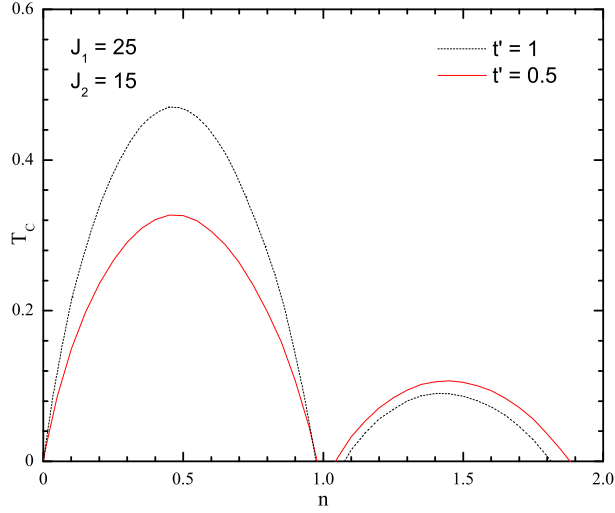


FIG. 6: (color online) Curie temperature T_C as a function of charge density n for $J_1 = 25$, $J_2 = 15$, with $t' = 1/2$ and $t' = 1$

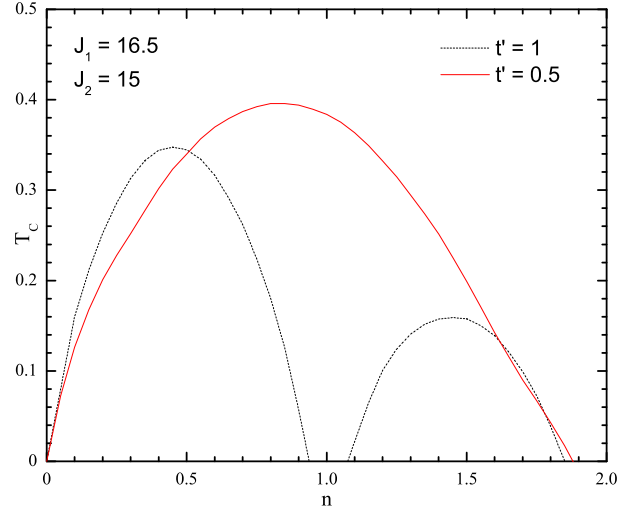


FIG. 8: (color online) Curie temperature T_C as a function of charge density n for $J_1 = 16.5$, $J_2 = 15$, with $t' = 1/2$ and $t' = 1$

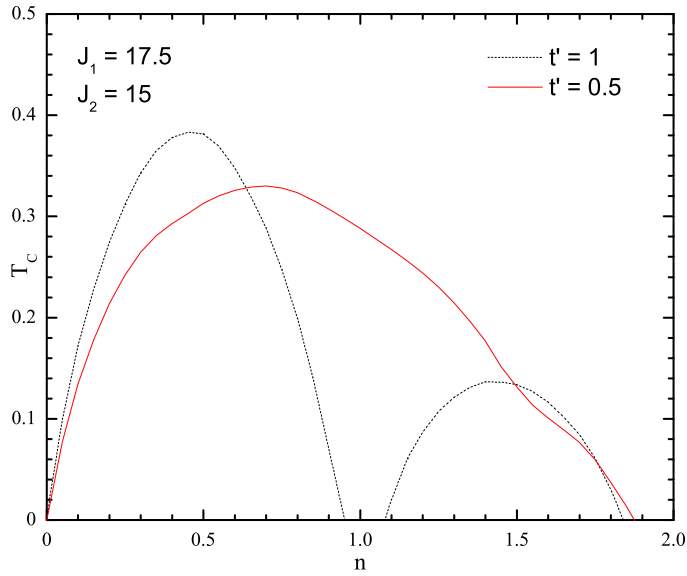


FIG. 7: (color online) Curie temperature T_C as a function of charge density n for $J_1 = 17.5$, $J_2 = 15$, with $t' = 1/2$ and $t' = 1$

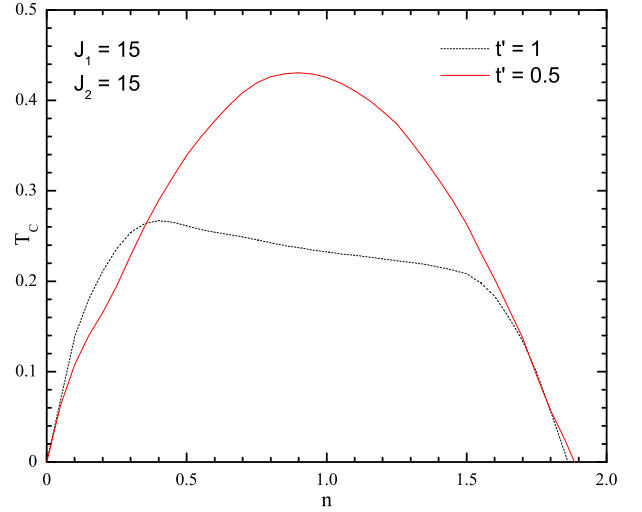


FIG. 9: (color online) Curie temperature T_C as a function of charge density n for the degenerated case $J_1 = 15$, $J_2 = 15$, with $t' = 1/2$ and $t' = 1$

portraits (Fig.1), where the instability of ferromagnetism near the charge carriers density $n = 2$ is evident. One can also note that decreasing the ratio J_1/J_2 leads to increase in the maximum value of $T_C(n)$.

The most important difference between our curves of critical temperature as a function of n and the ones in [14] is the lack of symmetry with respect to $n = 1$. This asymmetry originates from asymmetry in the phase diagrams, which in turn leads to asymmetry of the spin-stiffness curves.

We have also examined the effect of t' . Increasing its value results in enlargement of the ferromagnetic instability island around $n = 1$. The value of J_2^* decreases and the width of the island increases. This in turn leads to both quantitative and qualitative changes in $T_C(n)$ (see the black lines of Fig.6-9). An important conclusion is that Curie temperature increases with increasing t' if the bands are strongly split (Fig.6-7), while for weakly split bands we obtained an opposite behavior, the critical

temperature decreases (Fig.8-9).

V. SUMMARY

In summary, we have calculated the Curie temperature in two-band double exchange model. First we reduced the model to an effective Heisenberg type model for a vector which describes the local orientations of the total magnetization. Next, we use Schwinger-bosons mean-field theory to calculate the critical temperature. This technique of calculation is in agreement with Mermin-Wagner theorem, which means that employing our method of calculations for 2D system one obtains $T_C = 0$ [19, 20].

There are many quantitative and qualitative differences between our results and the results obtained within Dynamical Mean-Field Theory or Monte Carlo simulation approach. Maybe the most significant difference is that DMF and MC calculations lead to temperature's curves, as a function of fermion density, symmetric with respect to $n = 1$. This is not the result in the present paper. The asymmetry of the $T_C(n)$ curves with respect to $n = 1$ is a characteristic feature in our approach. This asymmetry is seen looking at the phase diagrams. It predetermines the asymmetry of the spin-stiffness curves which lead directly to the asymmetric $T_C(n)$ curves. The symmetry mentioned in the paper [14] is possible if the two bands are degenerated and Hund's constants are unphysically large.

Another important result reported in previous papers is that the critical temperature increases with increasing the interband hopping. Our calculations show that this is true when the band splitting is strong. If the bands split weakly, the assertion is opposite. The Curie temperature increases when the interband hopping decreases.

VI. ACKNOWLEDGMENTS

The authors acknowledge the financial support of the Sofia University, Project 037/2007

APPENDIX A: CALCULATION OF ρ

Here we present a detailed derivation of the spin stiffness constant ρ . We start from the two-band hamiltonian (II.1) and rewrite it in terms of Schwinger bosons (II.2) and spin-singlet fermions (II.5-II.6).

The resulting action is quadratic with respect to the spin-singlet fermions and one can integrate them out. To do so, it is convenient to represent the action as a sum of three terms:

$$S = S_f + S_{s-f}^{(1)} + S_{s-f}^{(2)} \quad (A.1)$$

where S_f is the free fermion action:

$$S_f = \int_0^\beta d\tau \left\{ \sum_{i\sigma} \left(\Psi_{i1\sigma}^+ \frac{\partial}{\partial \tau} \Psi_{i1\sigma} + \Psi_{i2\sigma}^+ \frac{\partial}{\partial \tau} \Psi_{i2\sigma} \right) + H_f \right\} \quad (A.2)$$

with free fermion hamiltonian H_f

$$\begin{aligned} H_f = & -s \sum_{il} J_l (\Psi_{il}^{+A} \Psi_{il}^A - \Psi_{il}^{+B} \Psi_{il}^B) - \mu \sum_{il\sigma} \Psi_{il\sigma}^+ \Psi_{il\sigma} \\ & - \sum_{\langle ij \rangle \sigma} t_1 (\Psi_{i1\sigma}^+ \Psi_{j1\sigma} + \Psi_{j1\sigma}^+ \Psi_{i1\sigma}) \\ & - \sum_{\langle ij \rangle \sigma} t_2 (\Psi_{i2\sigma}^+ \Psi_{j2\sigma} + \Psi_{j2\sigma}^+ \Psi_{i2\sigma}) \\ & - \sum_{\langle ij \rangle \sigma} t' (\Psi_{i1\sigma}^+ \Psi_{j2\sigma} + \Psi_{j2\sigma}^+ \Psi_{i1\sigma} + \Psi_{i2\sigma}^+ \Psi_{j1\sigma} + \Psi_{j1\sigma}^+ \Psi_{i2\sigma}) \end{aligned} \quad (A.3)$$

We remind of the notations $\Psi_{il1} = \Psi_{il}^A$ and $\Psi_{il2} = \Psi_{il}^B$, so that the sum over σ in the above equation is a sum over A and B . It is convenient to represent the term describing spin-fermion interaction as a sum of two terms:

$$\begin{aligned} S_{s-f}^{(1)} = & \int_0^\beta d\tau \left\{ -\frac{t_1}{2s} \sum_{\langle ij \rangle} \left[(\varphi_{i\sigma}^+ \varphi_{j\sigma} - 2s) (\Psi_{1i}^{+A} \Psi_{1j}^A + \Psi_{1j}^{+B} \Psi_{1i}^B) + (\varphi_{j\sigma}^+ \varphi_{i\sigma} - 2s) (\Psi_{1i}^{+B} \Psi_{1j}^B + \Psi_{1j}^{+A} \Psi_{1i}^A) \right] \right. \\ & - \frac{t_2}{2s} \sum_{\langle ij \rangle} \left[(\varphi_{i\sigma}^+ \varphi_{j\sigma} - 2s) (\Psi_{2i}^{+A} \Psi_{2j}^A + \Psi_{2j}^{+B} \Psi_{2i}^B) + (\varphi_{j\sigma}^+ \varphi_{i\sigma} - 2s) (\Psi_{2i}^{+B} \Psi_{2j}^B + \Psi_{2j}^{+A} \Psi_{2i}^A) \right] \\ & - \frac{t'}{2s} \sum_{\langle ij \rangle} \left[(\varphi_{i\sigma}^+ \varphi_{j\sigma} - 2s) (\Psi_{1i}^{+A} \Psi_{2j}^A + \Psi_{2i}^{+A} \Psi_{1j}^A + \Psi_{2j}^{+B} \Psi_{1i}^B + \Psi_{1j}^{+B} \Psi_{2i}^B) \right. \\ & \left. \left. + (\varphi_{j\sigma}^+ \varphi_{i\sigma} - 2s) (\Psi_{1i}^{+B} \Psi_{2j}^B + \Psi_{2i}^{+B} \Psi_{1j}^B + \Psi_{2j}^{+A} \Psi_{1i}^A + \Psi_{1j}^{+A} \Psi_{2i}^A) \right] \right\} \quad (A.4) \end{aligned}$$

$$\begin{aligned}
S_{\text{s-f}}^{(2)} = \int_0^\beta d\tau \Bigg\{ & -\frac{t_1}{2s} \sum_{\langle ij \rangle} \left[(\varphi_{i1}^+ \varphi_{j2}^+ - \varphi_{j1}^+ \varphi_{i2}^+) (\Psi_{1j}^{+A} \Psi_{1i}^B - \Psi_{1i}^{+A} \Psi_{1j}^B) + (\varphi_{i1} \varphi_{j2} - \varphi_{i2} \varphi_{j1}) (\Psi_{1i}^{+B} \Psi_{1j}^A - \Psi_{1j}^{+B} \Psi_{1i}^A) \right] \\
& -\frac{t_2}{2s} \sum_{\langle ij \rangle} \left[(\varphi_{i1}^+ \varphi_{j2}^+ - \varphi_{j1}^+ \varphi_{i2}^+) (\Psi_{2j}^{+A} \Psi_{2i}^B - \Psi_{2i}^{+A} \Psi_{2j}^B) + (\varphi_{i1} \varphi_{j2} - \varphi_{i2} \varphi_{j1}) (\Psi_{2i}^{+B} \Psi_{2j}^A - \Psi_{2j}^{+B} \Psi_{2i}^A) \right] \\
& -\frac{t'}{2s} \sum_{\langle ij \rangle} \left[(\varphi_{i1}^+ \varphi_{j2}^+ - \varphi_{j1}^+ \varphi_{i2}^+) (\Psi_{1j}^{+A} \Psi_{2i}^B + \Psi_{2j}^{+A} \Psi_{1i}^B - \Psi_{1i}^{+A} \Psi_{2j}^B - \Psi_{2i}^{+A} \Psi_{1j}^B) \right. \\
& \left. + (\varphi_{i1} \varphi_{j2} - \varphi_{i2} \varphi_{j1}) (\Psi_{2i}^{+B} \Psi_{1j}^A + \Psi_{1i}^{+B} \Psi_{2j}^A - \Psi_{1j}^{+B} \Psi_{2i}^A - \Psi_{2j}^{+B} \Psi_{1i}^A) \right] \Bigg\} \quad (\text{A.5})
\end{aligned}$$

To diagonalize the free fermion Hamiltonian H_f we switch to momentum space,

$$\begin{aligned}
H_f = & \sum_{lk} [\varepsilon_{kl}^A \Psi_{lk}^{A+} \Psi_{lk}^A + \varepsilon_{kl}^B \Psi_{lk}^{B+} \Psi_{lk}^B] \\
& + \sum_{k\sigma} [\varepsilon_k (\Psi_{1k\sigma}^+ \Psi_{2k\sigma} + \Psi_{2k\sigma}^+ \Psi_{1k\sigma})] \quad (\text{A.6})
\end{aligned}$$

where the dispersions for A and B fermions are

$$\begin{aligned}
\varepsilon_{kl}^A &= -2t_l \sum_k \cos k_\mu - sJ_l - \mu \\
\varepsilon_{kl}^B &= -2t_l \sum_k \cos k_\mu + sJ_l - \mu \\
\varepsilon(k) &= -2t' \sum_\mu \cos k_\mu
\end{aligned} \quad (\text{A.7})$$

The Hamiltonian is diagonalized by means of the transformation:

$$\begin{aligned}
\Psi_{1k}^A &= u_k^A f_{1k}^A + v_k^A f_{2k}^A & \Psi_{2k}^A &= -v_k^A f_{1k}^A + u_k^A f_{2k}^A \\
\Psi_{1k}^B &= u_k^B f_{1k}^B + v_k^B f_{2k}^B & \Psi_{2k}^B &= -v_k^B f_{1k}^B + u_k^B f_{2k}^B
\end{aligned} \quad (\text{A.8})$$

Solving the equations for u and v gives

$$u_k^R = \sqrt{\frac{1}{2}(1 + x_k^R)} \quad (\text{A.9})$$

$$v_k^R = \text{sign}(\varepsilon(k)) \sqrt{\frac{1}{2}(1 - x_k^R)}$$

$$\text{with} \quad x_k^R = \frac{\varepsilon_{2k}^R - \varepsilon_{1k}^R}{\sqrt{4\varepsilon^2(k) + (\varepsilon_{2k}^R - \varepsilon_{1k}^R)^2}} \quad (\text{A.10})$$

The resulting Hamiltonian is

$$\begin{aligned}
H_f = \sum_k \Bigg[& E_{1k}^A f_{1k}^{+A} f_{1k}^A + E_{1k}^B f_{1k}^{+B} f_{1k}^B \\
& + E_{2k}^A f_{2k}^{+A} f_{2k}^A + E_{2k}^B f_{2k}^{+B} f_{2k}^B \Bigg] \quad (\text{A.11})
\end{aligned}$$

with dispersions for the quasi-particles

$$\begin{aligned}
E_{1k}^A &= \frac{\varepsilon_{2k}^A + \varepsilon_{1k}^A}{2} - \frac{1}{2} \sqrt{4\varepsilon^2(k) + (\varepsilon_{2k}^A - \varepsilon_{1k}^A)^2} \\
E_{2k}^A &= \frac{\varepsilon_{2k}^A + \varepsilon_{1k}^A}{2} + \frac{1}{2} \sqrt{4\varepsilon^2(k) + (\varepsilon_{2k}^A - \varepsilon_{1k}^A)^2}
\end{aligned} \quad (\text{A.12})$$

$$\begin{aligned}
E_{1k}^B &= \frac{\varepsilon_{2k}^B + \varepsilon_{1k}^B}{2} - \frac{1}{2} \sqrt{4\varepsilon^2(k) + (\varepsilon_{2k}^B - \varepsilon_{1k}^B)^2} \\
E_{2k}^B &= \frac{\varepsilon_{2k}^B + \varepsilon_{1k}^B}{2} + \frac{1}{2} \sqrt{4\varepsilon^2(k) + (\varepsilon_{2k}^B - \varepsilon_{1k}^B)^2}
\end{aligned} \quad (\text{A.13})$$

To write the spin-fermion interaction $S_{\text{s-f}}$ in terms of the new fermions we introduce the notations

$$f_{k_1}^+ = (f_{1k_1}^{+A} \ f_{1k_1}^{+B} \ f_{2k_1}^{+A} \ f_{2k_1}^{+B}) \quad f_{k_2} = \begin{pmatrix} f_{1k_2}^A \\ f_{1k_2}^B \\ f_{2k_2}^A \\ f_{2k_2}^B \end{pmatrix} \quad (\text{A.14})$$

Now we rewrite the action in the form

$$S_{\text{s-f}} = \int_0^\beta d\tau_1 d\tau_2 \sum_{k_1 k_2} f_{k_1}^+ (\tau_1) W_{k_1 k_2} (\tau_1 - \tau_2) f_{k_2} (\tau_2) \quad (\text{A.15})$$

where the matrix $W_{k_1 k_2} (\tau_1 - \tau_2)$ is a sum of two terms

$$W_{k_1 k_2} (\tau_1 - \tau_2) = W_{k_1 k_2}^0 (\tau_1 - \tau_2) + W_{k_1 k_2}^{\text{int}} (\tau_1 - \tau_2). \quad (\text{A.16})$$

$W_{k_1 k_2}^0 (\tau_1 - \tau_2)$ is the free fermion action

$$W_{k_1 k_2}^0(\tau_1 - \tau_2) = \delta_{k_1 k_2} \delta(\tau_1 - \tau_2) \begin{pmatrix} \frac{\partial}{\partial \tau_2} + E_{1k_2}^A & 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial \tau_2} + E_{1k_2}^B & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial \tau_2} + E_{2k_2}^A & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial \tau_2} + E_{2k_2}^B \end{pmatrix} \quad (\text{A.17})$$

while $W_{k_1 k_2}^{\text{int}}(\tau_1 - \tau_2)$ is a sum of two matrixes, corresponding to $S_{\text{s-f}}^{(1)}$ and $S_{\text{s-f}}^{(2)}$

$$\begin{aligned} W_{k_1 k_2}^{\text{int}}(\tau_1 - \tau_2) = & \delta(\tau_1 - \tau_2) \begin{pmatrix} K_{k_1 k_2}(\tau_2) & 0 & 0 & 0 \\ 0 & L_{k_1 k_2}(\tau_2) & 0 & 0 \\ 0 & 0 & N_{k_1 k_2}(\tau_2) & 0 \\ 0 & 0 & 0 & O_{k_1 k_2}(\tau_2) \end{pmatrix} \\ & + \delta(\tau_1 - \tau_2) \begin{pmatrix} 0 & A_{k_1 k_2}(\tau_2) & 0 & B_{k_1 k_2}(\tau_2) \\ C_{k_1 k_2}(\tau_2) & 0 & D_{k_1 k_2}(\tau_2) & 0 \\ 0 & E_{k_1 k_2}(\tau_2) & 0 & F_{k_1 k_2}(\tau_2) \\ G_{k_1 k_2}(\tau_2) & 0 & I_{k_1 k_2}(\tau_2) & 0 \end{pmatrix} \end{aligned} \quad (\text{A.18})$$

with matrix elements

$$\begin{aligned} K_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_k \left(\sum_{\mu=1}^3 \frac{\cos k_\mu}{3} \right) \sum_{\langle ij \rangle} (\varphi_{i\sigma}^+ \varphi_{j\sigma} + \varphi_{j\sigma}^+ \varphi_{i\sigma} - 4s) \left[t_1 (u_k^A)^2 + t_2 (v_k^A)^2 - 2t' u_k^A v_k^A \right] \\ L_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_k \left(\sum_{\mu=1}^3 \frac{\cos k_\mu}{3} \right) \sum_{\langle ij \rangle} (\varphi_{i\sigma}^+ \varphi_{j\sigma} + \varphi_{j\sigma}^+ \varphi_{i\sigma} - 4s) \left[t_1 (u_k^B)^2 + t_2 (v_k^B)^2 - 2t' u_k^B v_k^B \right] \\ K_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_k \left(\sum_{\mu=1}^3 \frac{\cos k_\mu}{3} \right) \sum_{\langle ij \rangle} (\varphi_{i\sigma}^+ \varphi_{j\sigma} + \varphi_{j\sigma}^+ \varphi_{i\sigma} - 4s) \left[t_1 (v_k^A)^2 + t_2 (u_k^A)^2 + 2t' u_k^A v_k^A \right] \\ L_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_k \left(\sum_{\mu=1}^3 \frac{\cos k_\mu}{3} \right) \sum_{\langle ij \rangle} (\varphi_{i\sigma}^+ \varphi_{j\sigma} + \varphi_{j\sigma}^+ \varphi_{i\sigma} - 4s) \left[t_1 (v_k^B)^2 + t_2 (u_k^B)^2 + 2t' u_k^B v_k^B \right] \end{aligned} \quad (\text{A.19})$$

$$\begin{aligned} A_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_{\langle ij \rangle} (\varphi_{i1}^+ \varphi_{j2}^+ - \varphi_{j1}^+ \varphi_{i2}^+) (e^{-ik_1 r_j + ik_2 r_i} - e^{-ik_1 r_i + ik_2 r_j}) [t_1 u_{k_1}^A u_{k_2}^B + t_2 u_{k_1}^A v_{k_2}^B - t' u_{k_1}^A v_{k_2}^B - t' v_{k_1}^A u_{k_2}^B] \\ B_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_{\langle ij \rangle} (\varphi_{i1}^+ \varphi_{j2}^+ - \varphi_{j1}^+ \varphi_{i2}^+) (e^{-ik_1 r_j + ik_2 r_i} - e^{-ik_1 r_i + ik_2 r_j}) [t_1 u_{k_1}^A v_{k_2}^B - t_2 v_{k_1}^A u_{k_2}^B + t' u_{k_1}^A u_{k_2}^B - t' v_{k_1}^A v_{k_2}^B] \\ E_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_{\langle ij \rangle} (\varphi_{i1}^+ \varphi_{j2}^+ - \varphi_{j1}^+ \varphi_{i2}^+) (e^{-ik_1 r_j + ik_2 r_i} - e^{-ik_1 r_i + ik_2 r_j}) [t_1 v_{k_1}^A u_{k_2}^B - t_2 u_{k_1}^A v_{k_2}^B - t' v_{k_1}^A v_{k_2}^B + t' u_{k_1}^A u_{k_2}^B] \\ F_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_{\langle ij \rangle} (\varphi_{i1}^+ \varphi_{j2}^+ - \varphi_{j1}^+ \varphi_{i2}^+) (e^{-ik_1 r_j + ik_2 r_i} - e^{-ik_1 r_i + ik_2 r_j}) [t_1 v_{k_1}^A v_{k_2}^B + t_2 u_{k_1}^A u_{k_2}^B + t' v_{k_1}^A u_{k_2}^B + t' u_{k_1}^A v_{k_2}^B] \end{aligned} \quad (\text{A.20})$$

$$\begin{aligned}
C_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_{\langle ij \rangle} (\varphi_{i1} \varphi_{j2} - \varphi_{j1} \varphi_{i2}) (e^{-ik_1 r_i + ik_2 r_j} - e^{-ik_1 r_j + ik_2 r_i}) [t_1 u_{k_1}^B u_{k_2}^A + t_2 v_{k_1}^B v_{k_2}^A - t' u_{k_1}^B v_{k_2}^A - t' v_{k_1}^B u_{k_2}^A] \\
D_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_{\langle ij \rangle} (\varphi_{i1} \varphi_{j2} - \varphi_{j1} \varphi_{i2}) (e^{-ik_1 r_i + ik_2 r_j} - e^{-ik_1 r_j + ik_2 r_i}) [t_1 u_{k_1}^B v_{k_2}^A - t_2 v_{k_1}^B u_{k_2}^A + t' u_{k_1}^B u_{k_2}^A - t' v_{k_1}^B v_{k_2}^A] \\
G_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_{\langle ij \rangle} (\varphi_{i1} \varphi_{j2} - \varphi_{j1} \varphi_{i2}) (e^{-ik_1 r_i + ik_2 r_j} - e^{-ik_1 r_j + ik_2 r_i}) [t_1 v_{k_1}^B u_{k_2}^A - t_2 u_{k_1}^B v_{k_2}^A - t' v_{k_1}^B v_{k_2}^A + t' u_{k_1}^B u_{k_2}^A] \\
I_{k_1 k_2}(\tau_2) &= -\frac{1}{2s} \frac{1}{N} \sum_{\langle ij \rangle} (\varphi_{i1} \varphi_{j2} - \varphi_{j1} \varphi_{i2}) (e^{-ik_1 r_i + ik_2 r_j} - e^{-ik_1 r_j + ik_2 r_i}) [t_1 v_{k_1}^B v_{k_2}^A + t_2 u_{k_1}^B u_{k_2}^A + t' v_{k_1}^B u_{k_2}^A + t' u_{k_1}^B v_{k_2}^A]
\end{aligned} \tag{A.21}$$

where N is the number of lattice's sites. Integrating the fermions out we obtain the effective action S_{eff}

$$S_{\text{eff}} = -\ln \det W = -\text{Tr} \ln W \tag{A.22}$$

Using the properties of the logarithm

$$\begin{aligned}
\text{Tr} \ln W &= \text{Tr} \ln (W^0 + W^{\text{int}}) = \text{Tr} \ln W^0 \\
&+ \text{Tr} \ln \left(\mathbb{1} + (W^0)^{-1} W^{\text{int}} \right)
\end{aligned} \tag{A.23}$$

we rewrite the effective action in the form

$$S_{\text{eff}} = -\text{Tr} \ln \left(1 + (W^0)^{-1} W^{\text{int}} \right) \tag{A.24}$$

where the term $\text{Tr} \ln W^0$ doesn't depend on the Schwinger bosons and we have dropped it. Finally, we expand the effective action in powers of

$$X_{k_1 k_2}(\tau_1, \tau_2) = \sum_q \int d\tau [W_{k_1 q}^0(\tau_1, \tau)]^{-1} W_{q k_2}^{\text{int}}(\tau, \tau_2) \tag{A.25}$$

The result is

$$S_{\text{eff}} = -\text{Tr} X + \frac{1}{2} \text{Tr} X^2 + \dots \tag{A.26}$$

The inverse matrix $(W_{k_1 k_2}^0)^{-1}$ is given by

$$(W_{k_1 k_2}^0)^{-1}(\tau_1, \tau_2) = \begin{pmatrix} \delta_{k_1 k_2} S_{1k_1}^A(\tau_1 - \tau_2) & 0 & 0 & 0 \\ 0 & \delta_{k_1 k_2} S_{1k_1}^B(\tau_1 - \tau_2) & 0 & 0 \\ 0 & 0 & \delta_{k_1 k_2} S_{2k_1}^A(\tau_1 - \tau_2) & 0 \\ 0 & 0 & 0 & \delta_{k_1 k_2} S_{2k_1}^B(\tau_1 - \tau_2) \end{pmatrix} \tag{A.27}$$

where $S_{lk}^\sigma(\omega) = \frac{1}{-i\omega + E_{lk}^\sigma}$ ($\sigma = A$ or B , $l = 1$ or 2). Replacing (A.18) into (A.25), we end up with two terms for $X_{k_1 k_2}$, one which is diagonal $X_{k_1 k_2}^{(1)}$, and one with zero diagonal elements $X_{k_1 k_2}^{(2)}$. Hence, one obtains for the

trace of the matrix X

$$\text{Tr} X = \text{Tr} X^{(1)}. \tag{A.28}$$

where

$$X_{k_1 k_2}^{(1)}(\tau_1, \tau_2) = \begin{pmatrix} S_{1k_1}^A(\tau_1 - \tau_2) K_{k_1 k_2}(\tau_2) & 0 & 0 & 0 \\ 0 & S_{1k_1}^B(\tau_1 - \tau_2) L_{k_1 k_2}(\tau_2) & 0 & 0 \\ 0 & 0 & S_{2k_1}^A(\tau_1 - \tau_2) N_{k_1 k_2}(\tau_2) & 0 \\ 0 & 0 & 0 & S_{2k_1}^B(\tau_1 - \tau_2) O_{k_1 k_2}(\tau_2) \end{pmatrix} \tag{A.29}$$

and the first term in the effective action (A.26) is

$$S_{\text{eff}}^{(1)} = -\frac{1}{2s} \frac{1}{N} \sum_k \left(\sum_{\mu=1}^d \frac{\cos k_\mu}{d} \right) \int_0^\beta d\tau \sum_{\langle ij \rangle} (\varphi_{i\sigma}^+ \varphi_{j\sigma} + \varphi_{j\sigma}^+ \varphi_{i\sigma} - 4s) \left\{ 2t' \left[u_k^A v_k^A (n_{2k}^A - n_{1k}^A) + u_k^B v_k^B (n_{2k}^B - n_{1k}^B) \right] + t_1 \left[(u_k^A)^2 n_{1k}^A + (v_k^A)^2 n_{2k}^A + (u_k^B)^2 n_{1k}^B + (v_k^B)^2 n_{2k}^B \right] + t_2 \left[(u_k^A)^2 n_{2k}^A + (v_k^A)^2 n_{1k}^A + (u_k^B)^2 n_{2k}^B + (v_k^B)^2 n_{1k}^B \right] \right\} \quad (\text{A.30})$$

To calculate the contribution of $S_{\text{eff}}^{(1)}$ to the spin-stiffness constant ρ in (II.24) we use the Holstein-Primakoff representation for the Schwinger bosons

$$\begin{aligned} \varphi_{1i} &= \varphi_{1i}^+ = \sqrt{2s - \frac{s}{M} a_i^+ a_i} \\ \varphi_{2i} &= \sqrt{\frac{s}{M}} a_i \quad \varphi_{2i}^+ = \sqrt{\frac{s}{M}} a_i^+ \end{aligned} \quad (\text{A.31})$$

and keep the terms quadratic with respect to the magnons a_i, a_i^+ . The result is

$$\begin{aligned} \rho_1 &= \frac{1}{2M} \frac{1}{N} \sum_k \left(\sum_{\mu=1}^d \frac{\cos k_\mu}{d} \right) \left\{ 2t' \left[(u_k^A v_k^A (n_{2k}^A - n_{1k}^A) + u_k^B v_k^B (n_{2k}^B - n_{1k}^B)) \right] \right. \\ &\quad \left. + t_1 \left[(u_k^A)^2 n_{1k}^A + (v_k^A)^2 n_{2k}^A + (u_k^B)^2 n_{1k}^B + (v_k^B)^2 n_{2k}^B \right] + t_2 \left[(u_k^A)^2 n_{2k}^A + (v_k^A)^2 n_{1k}^A + (u_k^B)^2 n_{2k}^B + (v_k^B)^2 n_{1k}^B \right] \right\} \quad (\text{A.32}) \end{aligned}$$

Calculating the contribution of the second term in (A.26) to the effective hamiltonian (II.24) we account for the fact that X^1 matrix is quadratic with respect to

magnons, hence it doesn't contribute. The contribution comes from $S_{\text{s-f}}^{(2)}$ (A.5) which leads to the matrix $X^{(2)}$.

$$X_{k_1 k_2}^{(2)}(\tau_1, \tau_2) = \begin{pmatrix} 0 & S_{1k_1}^A(\tau_1 - \tau_2) A_{k_1 k_2}(\tau_2) & 0 & S_{1k_1}^A(\tau_1 - \tau_2) B_{k_1 k_2}(\tau_2) \\ S_{1k_1}^B(\tau_1 - \tau_2) C_{k_1 k_2}(\tau_2) & 0 & S_{1k_1}^B(\tau_1 - \tau_2) D_{k_1 k_2}(\tau_2) & 0 \\ 0 & S_{2k_1}^A(\tau_1 - \tau_2) E_{k_1 k_2}(\tau_2) & 0 & S_{2k_1}^A(\tau_1 - \tau_2) F_{k_1 k_2}(\tau_2) \\ S_{2k_1}^B(\tau_1 - \tau_2) G_{k_1 k_2}(\tau_2) & 0 & S_{2k_1}^B(\tau_1 - \tau_2) I_{k_1 k_2}(\tau_2) & 0 \end{pmatrix} \quad (\text{A.33})$$

After some algebra we arrive at the following representa-

tion of the second term in (A.26)

$$\begin{aligned} S_{\text{eff}}^{(2)} &= \int d\tau_1 d\tau_2 \sum_{k_1 k_2} \left[S_{1k_1}^A(\tau_1 - \tau_2) A_{k_1 k_2}(\tau_2) S_{1k_2}^B(\tau_2 - \tau_1) C_{k_2 k_1}(\tau_1) + S_{1k_1}^A(\tau_1 - \tau_2) B_{k_1 k_2}(\tau_2) S_{2k_2}^B(\tau_2 - \tau_1) G_{k_2 k_1}(\tau_1) \right. \\ &\quad \left. + S_{2k_1}^A(\tau_1 - \tau_2) E_{k_1 k_2}(\tau_2) S_{1k_2}^B(\tau_2 - \tau_1) D_{k_2 k_1}(\tau_1) + S_{2k_1}^A(\tau_1 - \tau_2) F_{k_1 k_2}(\tau_2) S_{2k_2}^B(\tau_2 - \tau_1) I_{k_2 k_1}(\tau_1) \right] \quad (\text{A.34}) \end{aligned}$$

Switching from imaginary time τ representation to fre-

quency ω representation we calculate the expressions in

small ω approximation. The result is a

$$S_{lk_1}^A(\tau_1 - \tau_2) S_{l'k_2}^B(\tau_1 - \tau_2) \simeq \delta(\tau_1 - \tau_2) \int \frac{d\omega}{2\pi} S_{lk_1}^A(\omega) S_{l'k_2}^B(\omega). \quad (\text{A.35})$$

Next we make a change of wave-vectors variables $k_1 =$

$q + \frac{1}{2}k$, $k_1 = q - \frac{1}{2}k$, and calculate the expressions in small wave-vector k approximation. The expression (A.34) calculated in small frequency and small wave-vector approximation has the form

$$S_{\text{eff}}^{(2)} = \int \frac{d\omega}{2\pi s^2} \frac{1}{N} \sum_q \left(\sum_{\mu=1}^3 \frac{\sin^2 q_\mu}{3} \right) \sum_{ij} \left[(\varphi_{i1}^+ \varphi_{j2}^+ - \varphi_{j1}^+ \varphi_{i2}^+) (\varphi_{i1} \varphi_{j2} - \varphi_{j1} \varphi_{i2}) \right] \quad (\text{A.36})$$

$$\left[S_{1q}^A(\omega) S_{1q}^B(\omega) (t_1 u_q^A u_q^B + t_2 v_q^A v_q^B - t' u_q^A v_q^B - t' v_q^A u_q^B)^2 + S_{1q}^A(\omega) S_{2q}^B(\omega) (t_1 u_q^A v_q^B - t_2 v_q^A u_q^B + t' u_q^A u_q^B - t' v_q^A v_q^B)^2 \right.$$

$$\left. + S_{2q}^A(\omega) S_{1q}^B(\omega) (t_1 v_q^A u_q^B - t_2 u_q^A v_q^B - t' v_q^A v_q^B + t' u_q^A u_q^B)^2 + S_{2q}^A(\omega) S_{2q}^B(\omega) (t_1 v_q^A v_q^B + t_2 u_q^A u_q^B + t' v_q^A u_q^B + t' u_q^A v_q^B)^2 \right]$$

Our third step is to express the products of the Green functions, in the above equation, in terms of the fermi function $n(E) = 1/(e^E + 1)$

$$\int \frac{d\omega}{2\pi} S_{1q}^A(\omega) S_{1q}^B(\omega) = \frac{n(E_{1q}^B) - n(E_{1q}^A)}{E_{1q}^B - E_{1q}^A} \quad (\text{A.37})$$

$$\int \frac{d\omega}{2\pi} S_{1q}^A(\omega) S_{2q}^B(\omega) = \frac{n(E_{2q}^B) - n(E_{1q}^A)}{E_{2q}^B - E_{1q}^A} \quad (\text{A.38})$$

$$\int \frac{d\omega}{2\pi} S_{2q}^A(\omega) S_{1q}^B(\omega) = \frac{n(E_{1q}^B) - n(E_{2q}^A)}{E_{1q}^B - E_{2q}^A} \quad (\text{A.39})$$

$$\int \frac{d\omega}{2\pi} S_{2q}^A(\omega) S_{2q}^B(\omega) = \frac{n(E_{2q}^B) - n(E_{2q}^A)}{E_{2q}^B - E_{2q}^A}. \quad (\text{A.40})$$

Finally we use the representation of the Schwinger bosons (A.31). To calculate the contribution to the spin-stiffness constant it is enough to keep only the quadratic terms with respect to magnons

$$\rho_2 = \frac{2}{M} \frac{1}{V} \sum_q \left(\sum_{\mu=1}^d \frac{\sin^2 q_\mu}{d} \right) \left[(t_1 u_q^A u_q^B + t_2 v_q^A v_q^B - t' u_q^A v_q^B - t' v_q^A u_q^B)^2 \left(\frac{n(E_{1q}^B) - n(E_{1q}^A)}{E_{1q}^B - E_{1q}^A} \right) + \right.$$

$$+ (t_1 u_q^A v_q^B - t_2 v_q^A u_q^B + t' u_q^A u_q^B - t' v_q^A v_q^B)^2 \left(\frac{n(E_{2q}^B) - n(E_{1q}^A)}{E_{2q}^B - E_{1q}^A} \right) +$$

$$+ (t_1 v_q^A u_q^B - t_2 u_q^A v_q^B - t' v_q^A v_q^B + t' u_q^A u_q^B)^2 \left(\frac{n(E_{1q}^B) - n(E_{2q}^A)}{E_{1q}^B - E_{2q}^A} \right) +$$

$$\left. + (t_1 v_q^A v_q^B + t_2 u_q^A u_q^B + t' v_q^A u_q^B + t' u_q^A v_q^B)^2 \left(\frac{n(E_{2q}^B) - n(E_{2q}^A)}{E_{2q}^B - E_{2q}^A} \right) \right] \quad (\text{A.41})$$

The spin-stiffness constant in the effective action (II.24) is a sum of the expressions (A.32) and (A.41)

$$\begin{aligned}
\rho = & \left\{ \frac{t_1}{2M} \frac{1}{V} \sum_k \left(\sum_{\mu=1}^d \frac{\cos k_\mu}{d} \right) \left[(u_k^A)^2 n_{1k}^A + (v_k^A)^2 n_{2k}^A + (u_k^B)^2 n_{1k}^B + (v_k^B)^2 n_{2k}^B \right] + \right. \\
& + \frac{t_2}{2M} \frac{1}{V} \sum_k \left(\sum_{\mu=1}^d \frac{\cos k_\mu}{d} \right) \left[(u_k^A)^2 n_{2k}^A + (v_k^A)^2 n_{1k}^A + (u_k^B)^2 n_{2k}^B + (v_k^B)^2 n_{1k}^B \right] + \\
& + \frac{t'}{M} \frac{1}{V} \sum_k \left(\sum_{\mu=1}^d \frac{\cos k_\mu}{d} \right) \left[u_k^A v_k^A (n_{2k}^A - n_{1k}^A) + u_k^B v_k^B (n_{2k}^B - n_{1k}^B) \right] + \\
& + \frac{2}{M} \frac{1}{V} \sum_k \left(\sum_{\mu=1}^d \frac{\sin^2 k_\mu}{d} \right) \left[(t_1 u_k^A v_k^B + t_2 v_k^A v_k^B - t' u_k^A v_k^B - t' v_k^A u_k^B)^2 \left(\frac{n_{1k}^B - n_{1k}^A}{E_{1k}^B - E_{1k}^A} \right) + \right. \\
& + (t_1 u_k^A v_k^B - t_2 v_k^A v_k^B + t' u_k^A u_k^B - t' v_k^A v_k^B)^2 \left(\frac{n_{2k}^B - n_{1k}^A}{E_{2k}^B - E_{1k}^A} \right) + \\
& + (t_1 v_k^A u_k^B - t_2 u_k^A v_k^B - t' v_k^A v_k^B + t' u_k^A u_k^B)^2 \left(\frac{n_{1k}^B - n_{2k}^A}{E_{1k}^B - E_{2k}^A} \right) + \\
& \left. + (t_1 v_k^A v_k^B + t_2 u_k^A u_k^B + t' v_k^A u_k^B + t' u_k^A v_k^B)^2 \left(\frac{n_{2k}^B - n_{2k}^A}{E_{2k}^B - E_{2k}^A} \right) \right] \Bigg\} \quad (\text{A.42})
\end{aligned}$$

[*] Electronic address: naoum@phys.uni-sofia.bg

- [1] E. Dagotto, *Nanoscale Phase Separation and Colossal Magnetoresistance* (Springer-Verlag, Berlin, 2003) and references therein.
- [2] C. Zener, Phys. Rev. **81**, 440 (1951).
- [3] H. Ohno, et al, Appl.Phys.Lett. **69**, 363 (1996).
- [4] S. Yunoki, J. Hu, A. L. Malvezzi, A. Moreo, N. Furukawa, and E. Dagotto, Phys. Rev. Lett. **80**, 845 (1998).
- [5] A. Chattopadhyay, A. J. Millis, and S. Das Sarma, Phys.Rev. **B64**, 012416 (2001).
- [6] Daniel P. Arovas, and Francisco Guinea, Phys.Rev **B58**, 9150 (1998).
- [7] D. Pekker, S. Mukhopadhyay, N. Trivedi and P. M. Goldbart, Phys.Rev **B72**, 075118 (2005).
- [8] M. Kagan, D. Khomskii, and M. Mostovoy, Eur. Phys. J. **B12**, 217 (1999).
- [9] Shun-Qing Shen and Z. D. Wang, Phys.Rev **B61**, 9532 (2000).
- [10] Takashi Hotta, Mohammad Moraghebi, Adrian Feiguin,

Adriana Moreo, Seiji Yunoki, and Elbio Dagotta, Phys. Rev. Lett., **90**, 247203 (2003).

- [11] H. Roder, R. R. P. Singh, and J. Zang, Phys.Rev **B56**, 5084 (1997).
- [12] N. Furukawa, J. Phys. Soc. Jpn., **64** 2754 (1995).
- [13] A. J. Millis, P. B. Littlewood, and B. I. Shraiman, Phys. Rev. Lett. **74**, 5144 (1995).
- [14] F. Popescu, C. Şen, and E. Dagotto, Phys. Rev. **B73**, 180404(R) (2006).
- [15] M. Stier and W. Nolting, Phys. Rev.**B75**, 144409 (2007).
- [16] N. D. Mermin and H. Wagner, Phys. Rev. Lett. **17**, 1133 (1966).
- [17] J. R. Schrieffer, and P.A. Wolf, Phys.Rev. **194**, 491 (1966).
- [18] E. L. Nagaev, Phys. Rev. **B58**, 827 (1998).
- [19] D. P. Arovas and A. Auerbach, Phys.Rev. **B38**, 316 (1988).
- [20] D. Yoshioka, J. Phys. Soc. Jpn. **58**, 32 (1989).